Errors in Estimating River Discharge from Remote Sensing based on Manning’s Equation

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Overview
- The Surface Water and Ocean Topography satellite mission will provide unprecedented mapping of water surface heights, slopes, areal extent, and their changes in time.
- The purpose of this study is to assess the accuracy of indirect streamflow estimates that would likely result from applying SWOT-based measurements in a simple slope-area approach (Manning’s equation).
- The slope-area method is considered a first-order method and was developed for use with ground-based observations. SWOT will contribute additional spatial information that is expected to improve these estimates.

Surface Water and Ocean Topography Mission
- Ka-band Radar Interferometry (KaRIN).
- Look angles limited to less than 4.5°; 2-60 km wide swaths.
- 22-day repeat cycle, 78° inclination; all rivers, lakes, reservoirs observed at least twice every 22 days.
- Will measure reach-averaged river properties to a high degree of accuracy (Table 1).

Table 1: Requirements for the accuracy of SWOT measurements (Rodriguez, 2009).

<table>
<thead>
<tr>
<th>Measurement</th>
<th>Required accuracy (1σ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water surface height</td>
<td>10 cm (Averaged over 1 km²)</td>
</tr>
<tr>
<td>Water surface slope</td>
<td>1 cm (Over 10 km downstream area)</td>
</tr>
<tr>
<td>Water surface areal extent</td>
<td>20% (For all rivers at least 100 m wide)</td>
</tr>
</tbody>
</table>

Derived Quantities
- Manning’s equation
  \[
  Q = \frac{1}{n} \cdot \frac{A}{R^{1/2}} S^{1/2} \]
- Assuming rectangular cross-section and width >> depth
  \[
  Q = \frac{1}{n} \cdot \frac{w}{R^{1/2}} \left( \frac{z_0 + d}{S} \right)^{3/2} \]
- In terms of SWOT observables
  \[
  Q = \frac{1}{n} \cdot \frac{w}{R^{1/2}} \left( \frac{z_0 + d}{S} \right)^{3/2} \]
- Assuming uniform flow, S = friction slope – water surface slope
- Manning’s roughness coefficient
- Hydraulic radius
- Wetted perimeter
- Water width
- Water depth
- Initial water depth
- Manning equation

Test Data: In Situ Reach-Averaged Observations
- Table 2. Summary statistics for 1038 in situ observations of streamflow and coincident hydraulic properties on 103 river reaches used for testing the error propagation. The largest river included is the Amazon River. Compiled by Bjerkli et al. (2003).

First Order Uncertainty Analysis
- We assume that Manning’s equation is:
  \[
  Q = \frac{1}{n} \cdot \frac{A}{R^{1/2}} S^{1/2} \]
- Where
  \[
  Q = \frac{1}{n} \cdot \frac{w}{R^{1/2}} \left( \frac{z_0 + d}{S} \right)^{3/2} \]
- C is the covariance matrix
  \[
  C = \begin{bmatrix}
  \frac{\partial Q}{\partial w} & \frac{\partial Q}{\partial z_0} & \frac{\partial Q}{\partial d} \\
  \frac{\partial Q}{\partial w} & \frac{\partial Q}{\partial z_0} & \frac{\partial Q}{\partial d} \\
  \frac{\partial Q}{\partial w} & \frac{\partial Q}{\partial z_0} & \frac{\partial Q}{\partial d}
  \end{bmatrix}
  \]
- If the terms are assumed to be independent, this becomes:
  \[
  \sigma_Q^2 = \sigma_w^2 \left( \frac{1}{n} \cdot \frac{A}{R^{1/2}} S^{1/2} \right)^2 \left( \frac{\partial Q}{\partial w} \right)^2 + \sigma_{z_0}^2 \left( \frac{1}{n} \cdot \frac{A}{R^{1/2}} S^{1/2} \right)^2 \left( \frac{\partial Q}{\partial z_0} \right)^2 + \sigma_d^2 \left( \frac{1}{n} \cdot \frac{A}{R^{1/2}} S^{1/2} \right)^2 \left( \frac{\partial Q}{\partial d} \right)^2
  \]

Monte Carlo Estimates of Error
- Figure 5. 1000 random perturbations to each observation were generated based on the distributions in box to right (with z_0=0.5z_e and Q was calculated by inserting these observations into Manning’s equation. Mean and standard deviation of relative error in Q were calculated from all 1,038,000. Errors were progressively added from upper left to bottom right.

Conclusions
- For the base case of Manning’s equation for 1-D channel flow, instantaneous discharge can be estimated with accuracies at or near 20% for most rivers wider than 100 m, assuming an improved estimation of n.
- Instantaneous discharge errors in this approach are highly sensitive to errors in total water depth. Estimating depth around low flows would help to limit these errors.
- This analysis depends strongly on the knowledge of error standard deviation and covariance. Additional work is needed to verify and improve estimates of the magnitude of these terms.
- In situ observational errors and the implications of knowledge of spatial extent during times of overbank flow should be considered in future work.
- Future efforts should seek to better understand the correlations between variables. Spatial and temporal sampling combined with continuity and other hydrodynamic assumptions should provide additional constraints not considered here.

Figure 1. Schematic of the SWOT instrument.
Figure 2. Distributions of hydraulic characteristics for rivers used in this study, excluding the Amazon River.
Figure 3. First order uncertainty assuming independent errors, as in Eqn. 6, based on 1038 observations, binned by width. An ideal case of n=6 is used in the part c.
Figure 4. First order uncertainty assuming a correlation of 1.0 between all terms in Eqn. 5, except that errors in h, w, and z_e were assumed independent.
Figure 5. 1000 random perturbations to each observation were generated based on the distributions in box to right (with z_0=0.5z_e and Q was calculated by inserting these observations into Manning’s equation. Mean and standard deviation of relative error in Q were calculated from all 1,038,000. Errors were progressively added from upper left to bottom right.
Figure 6. Comparison of results from Monte Carlo with only errors in slope, h, and bathymetry depending on assumed z_0.

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